ANNUAL VARIATIONS OF E–P CROSS SECTION
AND TROPOSPHERIC WESTERLY ACCELERATION*

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Abstract

This is one part of the series study on the forcing of waves on basic flow, in which the annual variations of E–P cross section are used to compare the wave–mean flow interactions in the Northern and Southern Hemispheres. Results show that in either hemisphere, wave–mean flow interaction is very strong in winter, and very weak in summer. External forcing source for planetary waves in the Southern Hemisphere appears to be rather weak not only in summer, but also in winter. It is pointed out that in the troposphere, since the mass circulation is strong, and since the static stability is small, in the dynamic equation, the inertial effect of the residual circulation becomes important in balancing that of the E–P flux divergence. Therefore, when the westerly acceleration in the troposphere is studied, both terms of E–P flux and residual circulation should be considered. It turns out to be more convenient to use the conventional Euler system to investigate the direct contribution of eddies to the mean flow.

Keywords: annual variation, E–P flux, residual circulation, westerly acceleration.

I. Introduction

In their study on the energy conversion associated with the orography-induced waves, Eliassen and Palm[1] found the relation between zonal mean momentum and the divergence of wave energy. Later, Andrews and McIntyre[2] derived the mean flow transformed equation, re-defined the so-called E–P flux and introduced the concept of residual circulation. Since then the E–P cross section has been used widely in atmospheric sciences. Especially in the middle atmosphere, this has become an effective tool in the analyses of wave propagation and wave–mean flow interactions. For example, it has been used by Shiotani and Hirotou[3] to compare the differences between the major and minor sudden warming in the stratosphere.

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During the past years, attempts have been made by some authors to use the E-P cross section to study the westerly acceleration in the troposphere. In such cases, attention should be called to the following problems. The first problem concerns the existence of massive water vapour in the troposphere. The applicability of the non-acceleration theorem to a moist atmosphere has been proved by Stone and Salustri\(^4\). The studies of Wu et al.\(^5\) show the existence of prominent differences in the E-P cross sections in different seasons between dry and moist atmospheric models, especially in the lowest tropospheric layers in the subtropical areas where the effects of wave energy flux on the mean westerlies in these two atmospheric models possess opposite signs. The second problem concerns the impacts of the mean meridional circulation on the zonal mean status of the atmosphere. In the middle atmosphere, since the mean mass flux is very weak, and since the vertical static stability is very large, the convergence/divergence of E-P flux can then describe the deceleration/acceleration of the zonal mean flow rather accurately. However, in the troposphere, the mean meridional circulation is very strong, and the static stability of the atmosphere is weak, hence the residual circulation must play an important role in the temporal variations of the zonal mean flow. Therefore, when the time evolutions of the zonal mean basic state of the atmosphere are studied, both the impacts of the E-P flux and the residual circulation should be considered. In our parallel research (see Ref. [51]), the first problem was emphasized. As a continuation to the study, in this part, in addition to the annual variation of the E-P cross section in the dry model, attention is also paid to the second problem, i.e. the application of the E-P cross section to the study of the zonal westerly acceleration in the troposphere. In Section II, some insights will be cast on the dynamics of the E-P cross section. The annual variations and the corresponding dynamic significance of the E-P cross section and residual circulation are discussed respectively in Sections III and IV. Some dynamical problems concerning the application of the E-P cross section to the westerly acceleration in the troposphere are emphasized in Section V. This is followed by a brief discussion on the analyzed results.

II. E-P Cross Section and Dynamic Features in January

In the quasi-geostrophic and spherical pressure coordinate system, the momentum equation, thermal wind relation, continuity equation and thermodynamic equation can be written as\(^6\)

\[
[u], - f\tilde{\varphi} = (a \cos \varphi)^{-1} \nabla \cdot E + \mathcal{F},
\]

\[
f_0[u]_\varphi - a^{-1} R^* \theta = 0,
\]

\[
(a \cos \varphi)^{-1}(\tilde{\varphi} \cos \varphi)_\varphi + (\tilde{\omega})_\varphi = 0,
\]

\[
[\theta]_\varphi + \Theta \tilde{\omega} = [Q_m].
\]

Here, \([ \ ]\) and \(*\) represent respectively the zonal mean and the deviation therefrom; \(R^* = \frac{R}{\rho} (p/p_0)^k\); \([\mathcal{F}]\) and \([Q_m]\), the sources of momentum and heat respectively; \(E\), the E-P flux defined by

\[
\]
\[ E = (E(\phi), E(\rho)) = a \cos \phi (-[\nu^* \nu^*], f_\theta \Theta \nu^* \theta^*), \] 
\[ \n \cdot E = \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (E(\phi) \cos \phi) + \frac{\partial}{\partial \rho} E(\rho). \] 

The residual meridional circulation appearing in (1) to (4) is defined by
\[
\begin{align*}
\vec{\omega} &= [\nu] - \Theta^{-1}[\nu^* \theta^*]_ho, \\
\vec{\omega} &= [\omega] + (a \cos \phi)^{-1} \Theta^{-1}[\nu^* \theta^*] \cos \phi.
\end{align*}
\]
and based upon the continuity equation (3), a stream function \( \tilde{\zeta} \) can be introduced for the residual meridional circulation:
\[
\begin{align*}
2 \pi a \gamma \cos \phi \vec{\omega} &= -a^{-1} \tilde{\zeta}, \\
2 \pi a \gamma \cos \phi \vec{\nu} &= \chi.
\end{align*}
\]

Eq. (1) shows that the zonal mean westerly deceleration (acceleration) may result from the convergence (divergence) of the E-P flux. When the meridional velocity \( \nu \) and an arbitrary quantity \( A \) are expanded to
\[
\begin{align*}
\begin{cases}
\nu^* &= \sum_{k=1}^{m} (C_k \cos k\lambda + S_k \sin k\lambda), \\
A^* &= \sum_{k=1}^{n} (C_k \cos k\lambda + S_k \sin k\lambda),
\end{cases}
\end{align*}
\]
where \( k \) is the zonal mean wavenumber, then the eddy transfer of \( A \) contained in (5) and (6) can be expressed in terms of the transfer of \( A \) by waves with different wave numbers:
\[
[v^* A^*] = \sum_{k=1}^{m} \frac{1}{2} (C_k C_k + S_k S_k).
\]

Since the eddy transfer in the wavenumber domain beyond 10 is very weak, the total eddy flux of \( A \) can be approximately decomposed to the transfer due to planetary scale waves and synoptic scale waves, i.e.
\[
[v^* A^*] \simeq \sum_{k=1}^{3} \frac{1}{2} (C_k C_k + S_k S_k) + \sum_{k=4}^{9} \frac{1}{2} (C_k C_k + S_k S_k).
\]

The data to be used in the study of the E-P cross section and its annual variations are adopted from the global general circulation statistics ranging from September 1979 to August 1984.

Fig. 1 shows the E-P cross section of the 5-year mean January. No matter whether it is for planetary scale waves, for synoptic scale waves, or for total waves, the general characters of the E-P cross section are similar: the E-P flux is divergent below 850 hPa and convergent in the free atmosphere above 850 hPa. Notice that in the quasi-geostrophic framework, the divergence of E-P flux is proportional to the zonal mean meridional transfer of potential vorticity, i.e.
\[
[v^* q^*] = (a \cos \phi)^{-1} \n \cdot E.
\]

Besides, the zonal mean eddy enstrophy equation
$$\left[ \frac{q^*}{2} \right] + [v^* q^*][q]_y = [q^* F^*] - [\nu] \left[ \frac{q^*}{2} \right]_y$$

(13)

demonstrates that at the latitude $\varphi_0$ where eddy activity centre ($[q^*]_y$ is maximum) is located (or in the middle troposphere $p \approx 500$ hPa), a conservative system possesses $[q^*]_y$ (or $[\nu]$) $= F^* = 0$, so that

$$\left[ \frac{q^*}{2} \right]_y \approx -[v^* q^*][q]_y, \quad \varphi = \varphi_0 \text{ or } p \approx 500 \text{ hPa},$$

(14)

therefore, eddy development must be accompanied by the eddy flux of potential vorticity down the gradient of the zonal mean vorticity field. On the other hand,
for a long-term mean, there exists the following simple relation between the source and the eddy flux of potential vorticity:

\[ [u^*q^*][q], \approx [q^*F^*], \varphi = \varphi_0 \text{ or } P \approx 500 \text{ hPa}. \]  

(15)

In the free atmosphere, usually \([q]\), is positive, then the southward eddy flux of potential vorticity implied by the convergence of E-P flux (see 12 shown in Fig. 1) means that dissipation of eddy potential vorticity enstrophy (including cascade) must be compensated for by the eddy potential vorticity transfer down the gradient of the zonal mean potential vorticity so as to maintain the quasi-stationary state of zonal perturbations. In other words, the downgradient eddy flux of potential vorticity implies that in the free troposphere, stationary waves experience the effects of dissipation.

Below 850 hPa, the divergence of E-P flux implies northward eddy transfer of potential vorticity. According to (15), the eddy enstrophy consumption indicated by the upgradient eddy transfer of potential vorticity must be compensated for by the generation of eddy enstrophy. This prominent feature of the E-P cross section indicates that one of the possible procedures for wave generation and propagation in the atmosphere is as follows: at the bottom layers of the atmosphere, perturbations excited by external sources transfer potential vorticity northwards, hence the zonal mean surface westerlies at middle and high latitudes are maintained\(^{[8,3]}\). During their upward propagation in the free atmosphere, these eddies experience nonlinear cascade and other kinds of dissipation; in order to get development or maintenance they should draw energy from the zonal mean field.

In other aspects, the general feature of the January E-P cross section is in agreement with that for the December, January and February mean (see Ref. [5]). For example, the E-P flux of planetary scale eddies in the northern troposphere appears as a canopy of coconut tree whose top penetrates the tropopause and enters the stratosphere, whereas this flux is very weak in the Southern Hemisphere. The E-P flux of synoptic scale eddies possesses comparable intensity in the two hemispheres, and looks more like an anvil which is reflected towards the subtropical tropopause during its upward propagation. In addition, the distribution of the divergence of E-P flux reveals that in January in the Northern Hemisphere, both planetary and synoptic scale eddies can strongly develop by drawing energy from the basic flow, whereas in the Southern Hemisphere only the synoptic scale eddies can develop appreciably via interacting with the mean flow, and the intensity of the divergence is weaker than in the Northern Hemisphere.

The distribution in the two hemispheres of the E-P cross section in July (figures not shown, see Fig. 2 of [5] for reference) is opposite to that in January. However, even in winter season in the Southern Hemisphere, the interaction between planetary waves and basic flow is rather weak too. This might be due to the fact that in the Southern Hemisphere, the external planetary-scale sources, such as orography and sea-land contrast, are not as strong as in the Northern Hemisphere.

### III. Annual Variation of the E-P Cross Section

Generally, at the height of 400 hPa, there exists a convergence centre of E-P
flux, its intensity and zonal distribution represents the general characteristics of wave-mean flow interaction. In the Southern Hemisphere, the convergence centres for planetary scale eddies are located to the south of 75°S every month; and those for synoptic scale eddies are stably maintained between 66°S and 72°S with very weak annual variation (figures not shown). In the Northern Hemisphere, the latitude locations of the convergence centres of E-P flux for planetary scale eddies, synoptic scale eddies and total eddies all exhibit consistent annual variations: they are at midlatitudes in winter, and shift to high latitudes in summer (figures not shown). This means that the area of vigorous wave development due to wave-mean flow interaction is further south in winter than in summer, and is in good agreement with general synoptic observations.

Figs. 2(a) and (b) represent the annual variation of the intensity of the convergence centre of E-P flux at 400 hPa. Its general feature for the two hemispheres is strong in winter and weak in summer. This feature is more obvious in the Northern Hemisphere: the convergence for the total eddies in December and January is 3 times as strong as in July, indicating waves being able to develop much more easily in winter than in summer. In the Southern Hemisphere, rather than in winter, the maxima in the annual variation appear in spring (September and October) and autumn (March and April). Since such variation is mainly from planetary scale eddies whose centres are located to the south of 75°S, and there data are scarce and with less accuracy, the above conclusion concerning the southern annual variation needs to be proved by further observations.

Notice that the convergence centres for different wave bands are located at different latitudes. Thus, Fig. 2 only shows the annual variation of the intensity of the convergence centre for individual wave band. The intensities between different wave bands cannot be compared with each other. In fact, due to the spherical geometry, the E-P flux term in the angular momentum equation should contain the additional factor \( \cos^2 \varphi \) (see Ref. [5]). In Fig. 2(b), therefore, the effects of the convergence centre for planetary scale eddies which is located in the southern polar area should

\[ I \propto (\cos \varphi)^{\pm \nu} \cdot E \]

Fig. 2. Annual variations in the intensity of the convergence centre of E-P flux, i.e. \((\cos \varphi)^{\pm \nu} \cdot E\), for different wave bands at 400 hPa in the two hemispheres. The heavy and light solid lines, and the dotted line denote respectively the variations for total eddies, planetary scale and synoptic scale eddies.

(a) Variations in the Northern Hemisphere; (b) variations in the Southern Hemisphere.
be much less than that for synoptic scale eddies which is at midlatitudes. The later
discussions will prove that in the Southern Hemisphere, the interactions with the
basic flow of synoptic scale eddies in midlatitudes are much stronger than those of
planetary scale eddies.

Fig. 3 shows the annual variation of the latitudinal distribution of the E-P
flux divergence for total eddies at the height of 400 hPa. Again, the wave-mean
flow interactions are stronger in winter than in summer. In the northern winter (Dec-
cember to February), the intensity of the maximum centre in the zone between 50°
N and 70° N reaches $-12 \times 10^{-2}$ m·s$^{-2}$, about 50% stronger than the corresponding
centre in the Southern Hemispheric winter (June to August). This is mainly due to
the contributions of planetary scale eddies (figures not shown). It is seen clearly that
the annual variation in the Southern Hemisphere is relatively small. In the Northern
Hemisphere, however, the E-P flux divergence for either planetary scale or synoptic
scale eddies possesses prominent annual variations: in the winter half year (September
to March), waves can develop via interacting with the basic flow over the vast
extratropical areas; whereas in the mid summer (July) wave-mean flow interaction is
only observed appreciably at high latitudes ($\varphi > 50^\circ$ N), and its intensity is only
about one third of that in winter; in the area of middle and low latitudes, it is
almost negligible. These reveal clearly the substantial difference in the dynamic fea-
tures of the Northern Hemisphere between winter and summer half years. In winter,
wave-mean flow interaction is very strong and dominates the wave development and
the variations of the basic flow. In summer, except in high latitude areas, such
atmospheric internal interaction is much weaker. Hence, the development of waves
and the variation of zonal mean flow in summer are to a great extent determined
by external sources (energy cascade may also play a role for individual wave develop-
ment).

Fig. 3. The annual variations of the latitudinal distribution of the E-P flux divergence
for total eddies at the height of 400 hPa. Isopleth intervals in $2 \times 10^{-2}$ m·s$^{-2}$.

Fig. 4 shows the annual variations of E-P flux divergence at 1000 hPa. As
pointed out in Section II, there exist external sources which can excite stationary
waves. Therefore, Fig. 4 provides a perspect to the annual variations of external wave
sources. In the planetary wave domain (Fig. 4(a)), in the Southern Hemisphere, except
for the weak centre at high latitudes in winter, the intensity of $\nabla \cdot \mathbf{E}$ is rather weak
in each month, indicating the weakness of external forcing source for planetary waves in the Southern Hemisphere (the data south to 60°S are not reliable). In the Northern Hemisphere, a strong divergence centre near 70°N is observed in each month. This is undoubtedly due to the $\beta$-effect. It is interesting to notice that in summer (June to August), another divergence centre is observed at about 50°N. This might result from the mechanical forcing of the Rocky Mountains and/or the strong land-sea contrast along the summer surface westerlies.

The E-P flux divergence areas for synoptic scale eddies are concentrated at
midlatitudes in the two hemispheres (Fig. 4(b)), and much stronger in the winter half year than in the summer half year. This hints that the lower tropospheric baroclinic instability generated by symmetric diabatic heating is very important for the production of eddy potential vorticity enstrophy at midlatitudes. It is the poleward eddy potential vorticity flux corresponding to the above physical processes which maintains the near-surface zonal mean westerlies in midlatitudes against surface frictions. This thermodynamic feature of the lower troposphere possesses stronger annual variations in the Northern Hemisphere: the divergence is weak in summer, and the strongest divergence does not occur in winter, but in late autumn (November) and early spring (April). It may be due to the fact that in the mid winter, the polar front and the sub-polar front are both very active. Vast areas of the Northern Hemisphere are under the control of polar or sub-polar cold air mass, so that the activities of baroclinic cyclones are reduced. In late autumn and early spring, the tropical warm air mass is relatively strong, its confrontation with the powerful polar cold air then leads to the formation of the strong baroclinic zones in midlatitudes.

The annual variation of the E-P flux divergence for total eddies near the surface (Fig. 4(c)) is slightly weaker in the Southern Hemisphere than in the Northern Hemisphere, and is determined mainly by synoptic scale eddies. In the Northern Hemisphere, this annual variation is very strong and is determined by both planetary and synoptic scale eddies. In the latitude zone between 40°N and 70°N, the maximum interaction centre appears from November to February. The summer divergence centre is weaker, and is mainly confined in the area near 50°N.

In Fig. 1(c), if we take a vertical column at 50°N of the vertical distribution of \((\cos \varphi)^{-1}\nabla \cdot \mathbf{E}\) and investigate its annual variation, we can get Fig. 5. At this latitude the eddy potential vorticity flux \(\mathbf{v}^* \mathbf{q}^*\) below 800 hPa is up the gradient of \([\mathbf{q}]\), perturbation can develop only via drawing energy from external sources, hence zonal mean surface westerly can be maintained. In the free atmosphere, one of the most prominent features is that the sign of \(\nabla \cdot \mathbf{E}\) between June and August is opposite to the other seasons. Since in the free atmosphere the external source of enstrophy is weak, the distribution of E-P flux divergence shown in Fig. 5 also implies that in

![Fig. 5. The annual variations of the vertical distributions of E-P flux divergence at 50°N. Isopleth intervals in 3 × 10^{-3} m \cdot s^{-1}.](image-url)
the areas of middle and high latitudes in summer, eddies hardly develop through drawing energy from the zonal basic fields. In other seasons, particularly between November and March, the wave-mean flow interactions in the Northern Hemisphere are very strong, and atmospheric perturbations can develop rather easily.

IV. Annual Variation of the Residual Circulation

In the transformed mean flow equation system (1) to (4), the thermodynamic

![Figure 6](image)

Fig. 6. The distribution of residual circulation $\tilde{X}$ and its annual variation. Units in $10^5$ ton/s.
(a) $\tilde{X}$ field in January; (b) the annual variation along 18°N; (c) the annual variation at 250 hPa.
equation (4) possesses quasi-Lagrangian form, therefore the upward branch ($\alpha < 0$) of the residual circulation indicates diabatic heating. Fig. 6(a) shows the distribution of the residual circulation $\mathbf{\xi}$ in January. Its general character is similar to that in northern winter (see Fig. 1d of [51]). One of the outstanding features of the $\mathbf{\xi}$ field calculated from the ECMWF data is the existence of the upper as well as the lower close circulation centre in tropical areas in the winter hemisphere in correspondence to the two separated upper and lower heating centres there. These two centres are located respectively at 250 hPa and 850 hPa. In the summer hemisphere, the upper centre disappears. In Fig. 6(b) is shown the annual variation of the vertical distribution of $\mathbf{\xi}$ at 18°N. From October to April, such double-layer-centre structure is obvious, whereas from May to September, with the influence of the northward-extending Southern Hemispheric positive circulation, the upper centre disappears, and only a weak lower centre is observed. For the latitudinal distribution of the $\mathbf{\xi}$ field at 250 hPa where the upper centre is located, its annual variation (Fig. 6(c)) shows that this upper centre exists from September to May in the Northern Hemisphere and from March to November in the Southern Hemisphere. The common feature is that it gets strongest in winter and disappears in summer. In transition seasons, it exists in both hemispheres, but comparatively weak. Since the upper Hadley circulation is related to the deep convections along the ITCZ, such annual variation of the intensity of the upper centre might be explained by the infinity of the Rossby deformation radius (proportioned to $f^{-1}$) at the equator so that deep convection can be much more easily excited when the location of the ITCZ is deviated from the equator. However, later research by Schneider and Lindzen[12] pointed out that in spherical geometry rather than in the f-plane approximation, the deformation radius near the equator approaches a constant equaling the geometric mean between the deformation radius at the pole and the earth's radius. Thus, CIISK at the equator can develop even more vigorously than in the sub tropics. Therefore, the fact that the intensity of the upper centre is stronger in winter than in transition seasons means that the formation of the upper centre is not only due to deep convections, but also related to the excitation effects on the secondary circulation of the eddy transfer properties. This is true at least for the formation of the Hadley circulation[12].

V. Problems Concerning Westerly Accelerations in the Troposphere

As stated in the introduction, the divergence of E-P flux can describe the relation between wave development and basic flow variation, and thus gets great success in the diagnoses of the sudden warming in the stratosphere. Encouraged by the success, in the recent years, some authors tried to apply this idea to analyzing the westerly variation in the troposphere. However, in some of the relevant studies, the impacts of the strong tropospheric mass circulation and the weak static stability were ignored so that the magnitude of westerly variation became very large, and sometimes even opposite in sign. Here, we will briefly discuss the problem by employing the climate mean E-P cross sections.

Notice that in Fig. 1(c) in mid-latitudes, the E-P flux is divergent below 800 hPa
and convergent above 800 hPa. From (1) we simply infer that the eddy effects lead to the tropospheric westerlies being accelerated in the bottom troposphere, and decelerated in the free atmosphere? From the definitions in (7) and the dynamic equation (1), it can be seen that part of eddy effects on the mean field is included in the term of residual circulation. The measure of static stability $\Theta_p$ in the troposphere is much smaller than in the stratosphere (about one quarter of the latter). Simple scale analyses then show a similar magnitude of the inertial term of residual

Fig. 7. The impacts of different factors on the zonal-mean westerly tendency $[\bar{u}]$. Units in $10^{-4}$ m s$^{-1}$ for (a) and (b), and $10^{-4}$ m s$^{-1}$ for (c).
(a) the impact of the residual circulation term $f \partial$; (b) the impact of momentum source $[\mathcal{E}]$; (c) the net direct eddy effects $-a^2 \cos^2 \varphi [(\bar{u} \times \cos \varphi)]_p$. 
circulation as that of the divergence term of E-P flux. Therefore, it is not appropriate to simply use the distribution of $\nabla \cdot E$ to determine the westerly variation in the troposphere. In Fig. 7(a) is shown the westerly variation due to the inertia effects of the residual circulation. In midlatitude areas, westerly is decelerated below 800 hPa and accelerated above 800 hPa, right on the opposite of the effects of E-P flux divergence (Fig. 1(c)). In addition, these two terms possess similar magnitude. Hence the two dominant terms in Eq. (1) are to a great extent cancelled by each other. Therefore, the sum of these two terms becomes a small residual as shown in Fig. 7(b). Since for monthly mean the local change rate of westerly is very weak, Fig. 7(b) basically represents the contribution of the $[\mathcal{F}]$ term to the westerly accelerations. Wu and Chen\cite{12} have estimated the impacts on westerly variation of external momentum sources calculated from the general circulation data\cite{12}. Their results (refer to their Fig. 4a) are in good agreement with those shown in Fig. 7(b). These demonstrate that it is the small residual between the residual circulation term and E-P flux term which balances the external momentum forcing and maintains the quasi-stationary state of the basic westerlies. Apparently, in the troposphere it is not appropriate to use only the E-P flux convergence to investigate the local westerly variation in disregard of the effects of the residual meridional circulation.

As a matter of fact, if (5) and (7) are substituted into (1), the compensation between the two dominant terms can be seen clearly. Actually, since

$$-f_0 \vec{\nu} = -f_0 [\nu] + f_0 (\Theta \vec{z}^* [\nu \theta^*])_r,$$

$$\frac{1}{a \cos \varphi} \nabla \cdot E = - \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \psi} [u^* \nu \cos^2 \psi] + f_0 (\Theta \vec{z}^* [\nu \theta^*])_r,$$

(1) is recovered to the following momentum equation in the Eulerian form:

$$[u], - f_0 [\nu] = - \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \psi} [u^* \nu \cos^2 \psi] + [\mathcal{F}].$$

The first term on the rhs is the direct effects of eddies on the westerly variations, and can be calculated from the general circulation statistics\cite{12}. Results shown in Fig. 7c (adopted from Fig. 1a of [13]) denote the intensification of midlatitude westerlies due to the inward eddy transfer of momentum from both low and high latitudes. Such westerly acceleration effects compensate for the surface dissipation of angular momentum, thus, the angular momentum balance of the atmosphere is maintained. The eddy transfer of momentum and heat can, of course, excite secondary circulation, and thereby affect westerly variation. However, as pointed out in our early study\cite{12}, this indirect effect is weaker than the direct effect.

In the free atmosphere in midlatitudes, the signs of westerly variation in Figs. 7(c) and 1(a) are opposite to each other. This seems to be contradictory. However, if their physical meanings are referred to, then such opposite tendency variations become natural. In Fig. 1(a), the negative tendency implies that during the eddy development energy conversion from the basic flow is required so that midlatitude westerlies are decelerated. Undoubtedly, this does not represent the total direct contribution
of eddies to the local westerly change. On the other hand, the positive tendency in Fig. 7c implies that eddy momentum transfer in the atmosphere can increase the midlatitude westerly momentum against frictional dissipation. When the local variation of westerlies in the troposphere is studied, the latter appears to be more adequate.

VI. Conclusions

The analyses of the annual variation of the E-P cross section reveal the fact that wave–mean flow interactions are very strong in the winter half year in the two hemispheres, especially in the Northern Hemisphere. The effects on the mean flow of synoptic scale eddies are mainly in midlatitudes, whereas those of planetary scale eddies are in high latitudes. In summer, the wave–mean flow interactions are very weak, the dynamic characters are very different from those in winter.

In midlatitudes, eddies excited by external sources near the surface transfer potential vorticity poleward and contribute to the maintenance of surface westerlies. In the free atmosphere, energy is supplied from the basic flow to eddies against dissipation. The concurrent equatorward eddy flux of potential vorticity decelerates zonal mean westerlies. In the Southern Hemisphere except in the midwinter, the external source of planetary scale waves is rather weak, and synoptic scale eddies dominate the transfer processes there.

The distribution of the residual circulation indicates the existence of upper and lower heating sources in the tropics respectively at 250 and 850 hPa. Corresponding to these sources, there exist upper as well as lower residual circulation centres. The upper centre exists only in the winter half year, reaches the strongest in winter, and disappears in summer.

In the troposphere, since mass circulation is strong, and since the static stability is small, the residual circulation contributes substantially to the zonal wind variations. This term and the term of the divergence of E-P flux are the two main terms in the momentum equation. Their small residual in the free atmosphere balances the source term so that the quasi-steady state of the basic flow is maintained. In the transformed mean flow equation system of Andrews et al.\textsuperscript{[2]}, the convergence of E-P flux in the free atmosphere in midlatitude troposphere only means that during the development of eddies, energy supply from basic flow is required. When the impacts of eddies on the local variation of tropospheric westerlies are studied, the Eulerian momentum equation is still the most basic form. The direct impacts of eddies are to intensify the midlatitude westerly, appearing as “negative viscosity”. It is such effect which resists the dissipation near the surface so that atmospheric angular momentum balance is maintained.

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