Analysis on observing optimization for the wind-driven circulation by an adjoint approach

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Abstract The adjoint approach is a variational method which is often applied to data assimilation widely in meteorology and oceanography. It is used for analyses on observing optimization for the wind-driven Sverdrup circulation. The adjoint system developed by Thacker and Long (1992), which is based on the GFDL Byrann-Cox model, includes three components, i.e. the forward model, the adjoint model and the optimal algorithm. The GFDL Byrann-Cox model was integrated for a long time driven by a batch of ideal wind stresses whose meridional component is set to null and zonal component is a sine function of latitudes in a rectangle box with six vertical levels and 2 by 2 degree horizontal resolution. The results are regarded as a “real” representative of the wind-driven Sverdrup circulation, from which the four dimensional fields are allowed to be sampled in several ways, such as sampling at the different levels or along the different vertical sections. To set the different samples, the fields of temperature, salinity and velocities function as the observational limit in the adjoint system respectively where the same initial condition is chosen for 4D VAR data assimilation. By examining the distance functions which measure the misfit between the circulation field from the control experiment of the adjoint system with a complete observation and those from data assimilation of adjoint approach in these sensitivity experiments respectively, observing optimizations for the wind-driven Sverdrup circulation will be suggested under a fixed observational cost.

Keywords: adjoint data assimilation, observing optimization, wind-driven circulation.

In the process of implementing Tropical Ocean and Global Atmosphere (TOGA), people come to realize that the development of ocean observation networks contributes greatly to the climatic monitoring[1]. At present, a number of international oceanic observing plans are being created and perfected, such as Tropical Atmosphere Ocean array (TAO)[2] and Pilot Research moored Array in the Tropical Atlantic (PIRATA). The problem ahead is how to optimize the observing scheme. Generally, this optimization means two aspects: one is how to choose the observing spots scientifically; the other is how to choose among the observing elements optimally in view of a certain type of oceanic movement or oceanic variation. According to the deep research of data assimilation in oceanography, how to optimize the oceanic observing scheme can be analyzed theoretically by the adjoint approach.
The adjoint approach is a new method which has been widely applied to the four-dimension variational (4D VAR) data assimilation \(^{[3-5]}\). It is realized by minimizing the functional built from the modeled values and the observed data. This optimal method sets up a trustworthy and objective connection between the model and the data, and it improves their mutual matching, in other words, based on the dynamic model one can assimilate the observing data, meanwhile also optimizing the dynamic models by the observing data. Thus, the adjoint method can retrieve some oceanic information which cannot be obtained by the present oceanic observing means.

In the last decade, many successful applications of the adjoint method have been made in oceanography\(^{[6]}\). So far, many oceanic adjoint models have come from the QG model\(^{[7]}\) or reduced gravity model\(^{[8]}\) which are of a little simple dynamics. Research of variational data assimilation of OGCM which is based on PE is just in the beginning and it is drawing considerable attention from oceanographers. Wang\(^{[9]}\) preliminarily introduced the performance of an adjoint system based on the Bryan-Cox oceanic circulation model, and did an optimal calculation on a non-steady oceanic initial field under the constraint of the ideal observing field.

In this paper, we used the adjoint system of the Bryan-Cox model to analyze the observing optimization for the wind-driven Sverdrup circulation. Through the long-term integration of the Bryan-Cox model\(^{[10,11]}\) we obtained the modeled wind-driven circulation and regarded it as a "true" oceanic Sverdrup circulation. We sampled the "true" sea on the different levels or along the different vertical sections and inserted these incomplete observations into the adjoint system. Then we obtained the distance between the modeled result from assimilation and the wind-driven Sverdrup circulation from the control experiment of the model with the complete observation. These distances can distinguish which is good or bad among the different observing schemes applied to the "true" ocean to do an incomplete "observation".

1 The adjoint assimilation system

1.1 Formulation of the system

The detailed description of the adjoint system used in this paper can be found in Wang's\(^{[9]}\). The mathematical-physical model comes from the work of Bergamasco\(^{[12]}\) and Yu\(^{[13]}\). Here we only give the simple mathematical expression.

Let the governing equation of the state vector \(x=(u,v,T,S)\) be

\[
\frac{dx}{dt} = F(x).
\]  

(1)

Suppose \(\delta x\) is a perturbation of solution \(x\). In the variational data assimilation, \(\delta x\) is regarded as the deviation between \(x_{\text{model}}\) (the model state) and \(x_{\text{obs}}\) (the corresponding observations).

Define a functional \(J\) by the general inner product to measure the distance between the model state and the observing state:

\[
J = \langle \delta x, \delta x \rangle
\]  

(2)
The tangent linear equation of (1) is
\[
\frac{d\delta x}{dt} = \left( \frac{\partial F}{\partial x} \right) \delta x. \tag{3}
\]

From (3), the evolution of delta \( \delta x \) in the time interval \([0,T]\) can be noted as
\[
\delta x(T) = R(0,T)\delta x(0), \tag{4}
\]
where \( R(0,T) \) is an operator which transfers the initial state of the ocean to its final state. Thus,
\[
J(T) = \langle \delta x(T), \delta x(T) \rangle \\
= \langle R(0,T)\delta x(0), R(0,T)\delta x(0) \rangle \\
= \langle R^*(T,0)R(0,T)\delta x(0), \delta x(0) \rangle, \tag{5}
\]
where \( R^*(T,0) \) is the adjoint operator of \( R(0,T) \). It evolves backward with time\(^{[14]}\).

One can derive the adjoint equation described by an adjoint operator through the Lagrange multiplier method. Form a Lagrange functional \( L \) as
\[
L = J + (\lambda, G), \tag{6}
\]
where \( G=0 \), it is eq. (1). \( \lambda = (\lambda_u, \lambda_v, \lambda_T, \lambda_y)^T \) is a group of Lagrange multipliers. The variational method is to find the optimal solution \( x^* \) of \( x \), which satisfies constraint of the following Euler-Lagrange equations:
\[
\begin{align*}
\frac{\partial L}{\partial x} &= 0, \tag{7a} \\
\frac{\partial L}{\partial \lambda_x} &= 0, \tag{7b} \\
\frac{\partial L}{\partial \lambda_{x_{\text{model}}}} &= 0, \tag{7c}
\end{align*}
\]
where we define the initial state of the model \( x_{\text{initial}} \) as the control variable of the adjoint system.

In other words, by modulating \( x_{\text{initial}} \), we aim to find the optimal initial value \( x^*_{\text{initial}} \), and let the model elements \( x_{\text{model}} \) integrating from this initial field be most identical with \( x_{\text{obs}} \).

In (7a) we meet the forward model again; from (7b) we can derive a set of equations about \( \lambda \) which is just the adjoint equation, i.e. the equations described by the adjoint operator \( R^*(T,0) \). (7c) is the criterion for the optimal solution. (7a)—(7c) constitute the adjoint system.

1.2 Model configuration

The calculation region is an ideal rectangular basin. There are 25 grids zonally and 20 meridionally. The south boundary is located at 14°N. In the vertical direction, it is divided into 6 levels, i.e. 50, 200, 500, 800, 1 000, 1 200 m depth respectively. The horizontal resolution is \( \Delta x = \Delta y = 2° \) and the time-step is \( \delta t=30 \) min. Set the wind stress as \( \tau_x = -\cos \left( \frac{j-2}{16} \pi \right) \), \( \tau_y = 0 \), where \( j \) is the grids in \( y \)-direction. The vertical viscosity coefficient \( A_v \) is 1 cm\(^2\) s\(^{-1}\), the horizontal viscosity coefficient \( A_n \) is 10\(^9\) cm\(^2\) s\(^{-1}\), the vertical heat and salinity diffusion coeffi-
cient $K_v$ is 1 cm$^2$·s$^{-1}$, the horizontal heat and salinity diffusion coefficient $K_h$ is 2×10$^7$ cm$^2$·s$^{-1}$. The weight $W_D$ is isotropic, for the speed $W_v=W_d=0.01$, for the temperature $W_T=100$, for the salinity $W_s=10^8$, the steady penalty coefficient $W'_{ua}=W'_{va}=W'_{TA}=W'_{TS}=10^{-2}$. The assimilation interval is 10 time-steps.

1.3 The procedure of the adjoint system

(a) Choose the first guess of the initial state properly, and integrate the dynamic model ten time-steps, then obtain the model element field $x=(u,v,T,S)$. Calculate the cost function $J$ and the deviation between the model element and the corresponding observing data.

(b) Force the adjoint equations to integrate backward by the deviation between the model element and the corresponding observing data, and calculate the gradient of Lagrange function $L$ with respect to the model initial state.

(c) Using the gradient, apply the descent algorithm to get a new model initial state so that the cost function $J$ tends to be smaller.

(d) According to whether the cost function $J$ is smaller than a critical value or not check whether the initial state is optimal, otherwise, do (a)–(c) again.

About the first guess for the adjoint data assimilation: Take the static ocean as the initial condition in which the elements of temperature and salinity are uniform horizontally and vertically (temperature is 4°C, and salinity is 0.034 9 standard unit), then force the oceanic circulation model to integrate 1 000 steps (about 21 days). There is Ekman drift in the surface which is driven by westerly in mid-latitudes and easterly in low latitudes (see fig. 1). On other levels, the circulation

![Fig. 1. The first guess for the adjoint data assimilation: currents in the six levels, unit of vector scale is cm/s.](image-url)
is weak anti-cyclonic. Near the west boundary, the phenomenon of western-strengthening in the circulation occurs. In view of its 3D structure, a barotropic mode prevails in the first guess.

About the "observing field" of the adjoint data assimilation: Take the field described above as the initial condition, and integrate the model 43 200 steps (900 model days, i.e. 2.5 model years). At that time the circulation field passed the over-shooting stage of adjustment and tended to be steady (fig. 2). It can be found that a steady Sverdrup circulation is established. This is the modeled response to the ideal wind stress which results in a baroclinic mode state, i.e. anti-cyclonic in the upper and cyclonic in the lower. To form the "observing field" over the assimilation interval, we go on integrating the model forward 10 steps and regard the model state in this period as a 4D "true sea" that may be sampled in several ways.

![Fig. 2. Same as fig. 1 but for the "observing field" of the adjoint data assimilation.](image)

The cost function is defined as

\[ J = J_{\text{data}} + J_{\text{steady}}, \]  

(8)

where

\[ J_{\text{data}} = \frac{1}{2} \int \left[ \left( u_{\text{model}} - u_{\text{obs}} \right)^2 + W_v \left( v_{\text{model}} - v_{\text{obs}} \right)^2 + W_T \left( T_{\text{model}} - T_{\text{obs}} \right)^2 + W_S \left( S_{\text{model}} - S_{\text{obs}} \right)^2 \right] dt. \]  

(9)

The above formula will be integrated in the 3D space and over the assimilation interval. Set the weight coefficients \( W_v, W_u, W_T, W_S \) as null at the grids that are not sampled.
\[ J_{\text{steady}} = \frac{1}{2} \left[ W'_u(u^N_{\text{model}} - u^0_{\text{model}})^2 + W'_v(v^N_{\text{model}} - v^0_{\text{model}})^2 \
+ W'_s(T^N_{\text{model}} - T^0_{\text{model}})^2 + W'_s(S^N_{\text{model}} - S^0_{\text{model}})^2 \right]. \]

The above formula, a steady penalty term, will be integrated in the 3D space, and it will introduce the constraint of the steady circulation into the adjoint system\(^{[12]}\).

2 Experiment schemes and results

The control experiment of data assimilation with complete observations:

We set the so-called complete observation about the above 4D “true sea”, i.e. sample the four variables \((u, v, T, S)\) by 4D space-time, and treat them as observing data for the adjoint system. The results show that the control experiment has a good ability to assimilate the barotropic mode in the ocean; it also can retrieve the baroclinic mode (the vertical shear with anticyclone in the upper and cyclone in the lower), however strength in the baroclinic mode is weaker compared with the “true sea”. It actually compresses the untrue strong current in the northeast of the rectangular oceanic area in the “true sea” (fig. 3).

![Diagram](image-url)  

Fig. 3. Same as fig. 1 but for the control experiment of data assimilation with complete observations.

The retrieval distance \(d\) of a sensitivity assimilation experiment is defined as the deviation between the field from this experiment and the wind-driven Sverdrup circulation from the control experiment with a complete observation. It expresses in terms of their least square. The retrieval distance in the baroclinic mode \(d_i\) is calculated from the baroclinic current components \(u^{\text{clinc}}\), \(v^{\text{clinc}}\) by deducting the vertical mean current:
\[ d_1 = \int \left[ (u_{\text{ctrl}} - u_{\text{sen}})^2 + (v_{\text{ctrl}} - v_{\text{sen}})^2 \right] d\sigma, \]  

(11)

where the subscript "ctrl" stands for the control experiment with complete observations and "sen" for a sensitivity experiment.

The retrieval distance in the barotropic mode \( d_2 \) comes from the barotropic stream function:

\[ d_2 = \int (\psi_{\text{ctrl}} - \psi_{\text{sen}})^2 d\sigma. \]  

(12)

The barotropic stream \( \psi \) can be diagnosed from the vertical mean barotropic current\(^{13} \).

Scheme 1: Sampling at the different levels

In this set of experiments, \( u, v, T, \) and \( S \) are sampled only at one level in the above 4D "true sea", and provide them as the observing data to the adjoint system. Since there are six levels in the model ocean, the retrieval distances in the baroclinic and barotropic modes in the six sensitivity experiments are shown in table 1.

<table>
<thead>
<tr>
<th>Sampled level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1/\text{cm}^2 \cdot \text{s}^2 )</td>
<td>549.18</td>
<td>605.04</td>
<td>822.74</td>
<td>897.85</td>
<td>936.03</td>
<td>759.75</td>
</tr>
<tr>
<td>( d_2/10^3 \text{Sv}^2 )</td>
<td>3.277</td>
<td>3.554</td>
<td>3.066</td>
<td>2.775</td>
<td>2.941</td>
<td>2.703</td>
</tr>
</tbody>
</table>

The retrieval effects for baroclinic modes are better in the upper levels, i.e. the retrieval distances of the upper levels are smaller than those of the lower levels. It is contrary to the barotropic modes. It indicates that the observing scheme should be suggested by possible theoretical analysis with respect to the different ocean motions in order to optimize observing. In these runs, the vertical difference of the retrieval effect in barotropic mode is not so obvious as that of baroclinic mode. With the definition of the cost function in eq. (9), in the cases of the same initial field, the stronger velocity the larger observing constraint, and the more it will improve the retrieval of the real ocean movement. It is embodied in the cases for baroclinic circulation in this trial, because the velocity in the upper level is larger than that in the lower, and the retrieval effect in the experiment with the upper observation is better than that with the lower one.

Considering the different field between the sensitivity experiments and the control experiment, a phenomenon can be identified as the local modified effect. In this sensitivity run, the anomaly in the current field at the sampled level is much smaller than those at the levels without observing constraints (fig. 4). This informs us that increasing the ocean observing information is very critical to improving the effect of ocean data assimilation. In the previous researches on ocean assimilation, in addition to the field observations, climatology or numerical model output of some elements were also thought of as observing-like information which will be inserted in the region without enough observations\(^{15} \).

Scheme 2. Section sampling experiment

In this scheme, \( u, v, T \) and \( S \) are sampled along three groups of meridional vertical sections
and are inserted respectively into the adjoint system as observing data. The first group are five meridional sections near the west boundary; the second are five sections in the middle area; the third are five sections near the east boundary.

In view of the retrieval effect of the baroclinic modes, the experiment with the middle group observations is better than the other two, followed by the one with observations near the west boundary. This is true as well of the retrieval of the barotropic modes (see table 2). Welandere’s theoretical analysis showed that in the inner area far from the frictional boundary, the wind-driven movement matches the Sverdrup balance; while near the east and west lateral frictional boundaries, the motions are more complex than those in the inner. This is why the retrieval effect with observation in the middle group is best. The physical reason which can explain the fact that the output of the experiment with the west group observations is better than that with the east is: the velocity near the west boundary is larger than the east, which is mentioned above.

<table>
<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
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<tr>
<td>---------</td>
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<tr>
<td>$d_1$/$\text{cm}^3 \cdot \text{s}^{-1}$</td>
</tr>
<tr>
<td>$d_210^{-8}\text{Sv}$</td>
</tr>
</tbody>
</table>

In the difference in the velocity speed between the sensitivity experiment and the control experiment (fig. 5), the local modified effect mentioned in Scheme 1 also appears. This trial also embodies another phenomenon, which is called the neighborhood modified effect. For example, in the experiment with the middle group observing, in addition to the local modified effect, the difference at more than one section east or west of the observing area is reduced to some extent while compared with those in the other two experiments. The fact shows that the adjoint data assimilation can dynamically regulate the local observations through the ocean circulation model and its adjoint model, which is transferred to its neighborhood spatially.
3 Conclusion

It is a complex problem to optimize oceanic observing schemes. Work in this paper is very preliminary. Only for the oceanic wind-driven Sverdrup circulation which is a simple large-scale oceanic movement and under the ideal conditions (a rectangle basin and an ideal wind-stress), do we analyze the optimization of the oceanic observing schemes by the adjoint approach. Our target is to propose an objective method for observing optimization different from the traditional method based on subjective and empirical thoughts.

Two phenomena are found in the sensitivity experiments. One is the local modified effect, i.e. so far as the difference in the current between the sensitivity experiment and the control experiment, the anomaly at the level or section with the observing constraint is much less than that at the level or section without the observing constraint. The local observing constraint also affects the circulation near the observing area, and makes its deviation from the circulation produced by the control experiment reduce. This is the so-called neighborhood modified effect.

The series of experiments in this paper show that the theoretical analysis of observing plan (such as taking the numerical experiment including data assimilation) is very necessary to form the optimized observing scheme for this kind of ocean movement. This optimization of ocean ob-
serving scheme can improve the effect and control the cost of the ocean observation.

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