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Severe weather diagnosis from the perspective of generalized slantwise vorticity development

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2.1 Introduction

In a frictionless and adiabatic dry atmosphere, the Ertel potential vorticity \( (PV_e) \) is conserved (Ertel, 1942). Many applications of the Ertel potential vorticity to the diagnosis of atmospheric motion were summarized by Hoskins et al. (1985). Based on the conservation of Ertel potential vorticity, Wu and Liu (1997) proposed a theory of slantwise vorticity development (SVD) to interpret the development of the vertical vorticity of a Lagrangian particle sliding down a slantwise isentropic surface. Many applications of the SVD theory have obtained reasonable results and demonstrated the development of vertical vorticity on the slope of an isentropic surface (Cui et al., 2002, Ma et al., 2002, Chen et al., 2004, Jiang et al., 2004, Wang et al., 2007). However, vertical vorticity development depends not only on the horizontal component of the \( PV_e \), but also on the static stability \( \theta_z \) under the conservation of \( PV_e \). More importantly, the latent heat release associated with precipitation plays a significant role in vertical vorticity development (Shen et al., 1986, Ding and Lu, 1990, Chen et al., 1996). Therefore, understanding how diabatic heating contributes to vertical vorticity development from the viewpoint of potential vorticity and diabatic heating \( (PV-Q) \) is important, and is one of the objectives of this study.

On the other hand, during some stages of vortex development, the adiabatic processes that are associated with the internal thermal structure of the atmosphere, and can be interpreted by SVD, can play comparable roles to diabatic heating during vortex development. Actually, SVD usually occurs during the initial stage of vortex development and before precipitation begins. After vortex development begins, vertical rising is generated within the vortex and precipitation appears, leading to condensation diabatic heating, and thereby enhancing the development of the vortex. In this regard, it is important and useful to understand how SVD occurs in a more general sense from the \( P-Q \) perspective, and this is another objective of this study.

The Tibetan Plateau (TP) vortex (TPV) is a shallow low-level cyclonic system that develops near 500 hPa, over the TP and in the lower troposphere over the plain areas. The TPV occurs during the boreal summer. In general, it originates over the central western TP, then moves eastward before dying out over the eastern TP. Its horizontal and vertical scales are typically about 500 and 2–3 km, respectively. Sometimes, the TPV persists and moves eastward out of the TP, and this often results in severe weather over eastern China, especially in the Sichuan Basin (Ye and Gao, 1979, Tao and Ding, 1981, Qiao and Zhang 1994, Li, 2002). Previous studies have shown that the formation of the TPV is driven by surface sensible heating, topography, static stability, boundary layer friction, and large-scale circulation of the mid-latitude trough or cyclone (Group of Tibetan Plateau Low System, 1978, Tao, 1980, Zhang et al., 1988, Ding, 1993, Chen et al., 1996, Ding, 2005). It is generally believed that the development of the TPV is closely related to diabatic heating, especially condensation heat release (Shen et al., 1986, Ding and Lu, 1990, Chen et al., 1996). The movement of the TPV depends on the difference in divergence between 200 and 500 hPa (Liu and Fu, 1985) and the steering flow of the southwesterly jet at 300 hPa (Qiao, 1987, Sun and Chen, 1988). However, there are still questions regarding TPV development. For example, how does the 3D heterogeneous distribution of the diabatic heating affect the development and movement of the TPV? What is the relationship between diabatic heating and circulation configuration during the development of the TPV? What role do adiabatic processes play in the development and movement of the TPV? These questions remain unanswered and need to be investigated. An attempt to address these points using the theory of generalized slantwise vorticity development from both \( PV-Q \) and \( PV-\theta \) perspectives is the main aim of this study.
2.2 Generalized slantwise vorticity development

By definition, $PV_e$ (Ertel, 1942) is expressed as

$$PV_e = a(2\Omega + \nabla \times V) \cdot \nabla \theta = \eta_a \cdot \nabla \theta,$$

(2.1)

where $\eta_a = \eta_i + \eta_j + \eta_k$ is the absolute vorticity per unit mass, and $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$ is the 3D gradient operator. Defining the horizontal component of $PV_e$ as

$$PV_2 = \eta_i \partial x + \eta_j \partial y = \eta \cdot \partial,$$

(2.2)

where $\eta = \eta_i + \eta_j$ is the horizontal vorticity, $\partial = \partial_x + \partial_y \nabla \theta$ is the horizontal gradient of the potential temperature, and $\nabla_z = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j$ is a horizontal gradient operator, then

$$\eta_z = \frac{PV_e - PV_2}{\partial_z} = \frac{PV_e}{\partial_z} - C_D, \quad \eta_z \neq 0,$$

(2.3)

where

$$C_D = \frac{PV_2}{\partial_z} = \eta \cdot \partial, \quad \eta_z \neq 0.$$

(2.4)

The notations used here are as follows: $\eta_i = a(\partial \Omega/\partial x), \eta_j = a(f + \nabla \phi - \partial \Omega/\partial y), \eta_k = \partial \Omega/\partial z, \eta = \partial \Omega/\partial z, \partial_x = \partial / \partial x, \partial_y = \partial / \partial y, \partial_z = \partial / \partial z$.

$\eta = 2\Omega \cos \phi$, and $f = 2\Omega \sin \phi, \Omega = 7.292 \times 10^{-4}$ is the rotation rate of the Earth, $\phi$ is latitude, $\theta$ is potential temperature, and $\alpha$ is specific volume.

To evaluate the development of vertical vorticity, taking the operator $\partial_z$ in Eq. (2.1) leads to

$$\frac{D\eta_z}{Dt} = \partial_z \left( \frac{PV_e - PV_2}{\partial_z} \right) = \frac{1}{\partial_z} \left( \frac{DPV_e}{Dt} - \frac{1}{\partial_z} \frac{DPV_2}{Dt} \right),$$

(2.5)

where $\partial_z = \frac{\partial}{\partial z} + V \cdot \nabla$. This demonstrates that the change in $PV_2$, $PV_e$, and $\theta_z$ along a Lagrangian particle track can influence the individual change in vertical vorticity of the particle.

2.2.1 Revisiting slantwise vorticity development

Based on the conservation of Ertel potential vorticity, Wu and Liu (1997) proposed the SVD theory to interpret the intense vertical vorticity development of a Lagrangian particle sliding down a slantwise isentropic surface. The condition for the SVD is also given in Wu and Liu (1997) and is duplicated here:

$$C_D(t + \Delta t) = C_D(t) < PV_e \left[ \frac{1}{\theta(t + \Delta t)} - \frac{1}{\theta(t)} \right].$$

(2.6)

The SVD for a statically stable atmosphere was explained schematically by Wu and Liu (1997) for a down-sliding case, and by Cui et al. (2003) for an up-sliding case.

Many applications of the SVD theory used to diagnose the occurrence of torrential rain and severe weather obtained reasonable results and demonstrated that vertical vorticity can develop dramatically along the sharply sloping isentropic surface, and the condition $C_D < 0$ was diagnosed in some cases (Cui et al., 2002, Ma et al., 2002, Chen et al., 2004, Jiang et al., 2004, Wang et al., 2007). However, as shown in Eq. (2.3), the vertical vorticity development depends not only on $C_D$, but also on the static stability $\theta_z$ under the conservation of $PV_e$. From the definition of $C_D$ in Eq. (2.4), a negative $C_D$ under stable stratification implies a negative $PV_2$. Hoskins et al. (1985) proved that in pressure coordinates, $PV_e^2$ is negative for geostrophic balance. In height coordinates, the hydrostatic balance yields

$$\begin{align*}
\frac{1}{\rho} \nabla \rho \Pi = \theta \nabla \Pi, \\
\frac{\partial \Pi}{\partial z} &= \frac{-g}{\theta} \partial\Pi/\partial z,
\end{align*}$$

(2.7)

where $\Pi = C_p(\xi^2 + \zeta \zeta^2)$ is the Exner function. Then, the geostrophic wind in the height coordinates is

$$\mathbf{V} = \frac{k}{\rho f} \times \nabla \rho = \theta \frac{k}{\rho f} \times \nabla \Pi.$$

(2.8)

Taking $\partial \Pi/\partial z$ upon Eq. (2.8) yields the thermal wind,

$$\partial V^z / \partial z = \frac{\partial \theta}{\partial z} \frac{k}{\rho f} \times \nabla \Pi + \theta \frac{k}{\rho f} \times \nabla \Pi = \eta_z \frac{k}{\rho f} \times \nabla \Pi + \frac{g}{\theta} \partial \Pi / \partial z = \theta \frac{k}{\rho f} \times \nabla \Pi + \frac{g}{\theta} \partial \Pi / \partial z.$$

(2.9)

Therefore,

$$PV_2 \approx \frac{1}{\rho} \left( \frac{k}{\rho f} \times \frac{\partial V^z}{\partial z} \right) \cdot \nabla \theta = -\frac{g}{\theta} \left( \theta \nabla \Pi \cdot \nabla \theta + \frac{g}{\theta} |\nabla \theta|^2 \right),$$

(2.10)

and

$$C_D = \frac{PV_2}{\partial_z} = -\frac{a}{\theta} \left( \nabla \Pi \cdot \nabla \theta + \frac{g}{\theta} |\nabla \theta|^2 \right).$$

(2.11)

In the Northern Hemisphere, $f$ is positive, and the second term $PV_2^2 = -\frac{g}{\theta} |\nabla \theta|^2$ on the right-hand side of Eq. (2.10) is negative. Although, the first term, $PV_2^2 = -\frac{g}{\theta} \nabla \Pi \cdot \nabla \theta$, is not necessarily negative, its magnitude is usually smaller than the second term, $PV_2^2$, so $PV_2$ is almost negative in the troposphere under the geostrophic balance. Figure 2.1 contains the 3D distributions of $C_D$ and $PV_2$ under real weather conditions at 0600 UTC 22 July 2008. The distributions for other time slices are similar. Figure 2.1 demonstrates that both $C_D$ and...
are usually negative throughout the troposphere. Therefore, using $C_D < 0$ alone as the criterion for SVD is inappropriate. For more general cases, the influence of the change in static stability needs to be considered. The SVD theory (Wu and Liu, 1997) is based on the conservation of Ertel potential vorticity, so it did not explicitly consider the impacts of diabatic heating. Recently, Wu et al. (2013) and Zheng et al. (2013) have proposed an approach to severe

![Diagram](image-url)
Severe weather diagnosis from the perspective of generalized slantwise vorticity development

weather diagnosis in eastern China based on generalized slantwise vorticity development. In the following two sections, generalized slantwise vorticity development (GSVD), as an extension of SVD, is used to consider both diabatic and adiabatic processes from a Lagrangian perspective.

2.2.2 Diabatic vorticity development

The first term on the right-hand side of Eq. (2.5) associated with diabatic heating is determined by the static stability $\theta_z$ and the change in $PV_e$:

$$\left(\frac{\partial PV_e}{\partial t}\right)_Q = \frac{1}{\theta_z} \frac{\partial PV_e}{\partial \theta}, \quad \theta_z \neq 0.$$  \hspace{1cm} (2.12)

Based on the potential vorticity equation (Ertel, 1942, Hoskins et al., 1985)

$$\frac{DPV_e}{Dt} = \eta_s \cdot \nabla Q + (\alpha \nabla \times F) \cdot \nabla \theta,$$  \hspace{1cm} (2.13)

where $Q = \frac{D\theta}{Dt}$ denotes diabatic heating. In the free atmosphere, friction $F$ can be neglected, so

$$\frac{DPV_e}{Dt} = \eta_s \cdot \nabla Q = \eta_s \cdot \frac{\partial Q}{\partial z} + \eta_s \cdot \nabla_s Q = \frac{DPV_e}{Dt_z} + \frac{DPV_e}{Dt_s},$$ \hspace{1cm} (2.14)

where

$$\frac{DPV_e}{Dt_z} = \eta_z \frac{\partial Q}{\partial z}$$ \hspace{1cm} (2.15)

and

$$\frac{DPV_e}{Dt_s} = \eta_s \cdot \nabla_s Q$$ \hspace{1cm} (2.16)

represent contributions to the $PV_e$ changes due to the non-uniformity of diabatic heating in the vertical and horizontal directions, respectively.

Impacts on vertical vorticity development of the vertical non-uniformity of diabatic heating

In the case of the non-uniformity of diabatic heating in the vertical direction, according to Eq. (2.15), $PV_e$ increases (decreases) where the vertical gradient of diabatic heating is positive (negative: Wu and Liu, 2000). This is illustrated schematically in Fig. 2.2a. In such circumstances, the vertical gradient of diabatic heating leads to positive (negative) $PV_e$ generation below (above) the maximum of the diabatic heating. According to Eq. (2.12), for a stable stratified atmosphere, the low-level positive $PV_e$ generation corresponds to an increasing cyclonic circulation that can strengthen the low-level vortex; and the upper-level negative $PV_e$ generation corresponds to an increasing anticyclone circulation that can weaken the upper-level vortex, causing the vortex to be confined in a lower vertical extension.

It should be noted that the condensation diabatic heating in a vortex usually occurs on its eastern side. This is because the horizontal advection of vorticity is negligible across the center of the vortex. Consequently, the vorticity equation at a steady state can be approximated as $\beta v = f \frac{\partial w}{\partial z}$ and the vertical velocity becomes $w \propto -f \frac{\partial h}{\partial z}$. As on the eastern (western) side of a vortex, $\frac{\partial h}{\partial z} < 0$ ($\frac{\partial h}{\partial z} > 0$), air rising (sinking) occurs normally on the eastern (western) side of the vortex. This implies that condensation diabatic heating usually occurs on the eastern side of a vortex. Consequently, the generation of positive vorticity in the lower troposphere on the east of a vortex associated with the vertical non-uniformity of condensation diabatic heating will contribute to the eastward movement of the vortex.

Fig. 2.2: Lagrangian $PV_e$ generation associated with (a) the vertical profile of diabatic heating $Q$ (thick solid line) and (b) the horizontal distribution of diabatic heating $Q$ (shading). Long solid arrow in (b) is the vertical shear ($\frac{\partial w}{\partial z}$) of horizontal wind, long dash arrow in (a) is the vertical vorticity $\eta_z$, and in (b) is the horizontal vorticity $\eta_s$, and short dot arrows are the vertical gradient ($\frac{\partial Q}{\partial z}$) of diabatic heating in (a) and $\nabla_s Q$ in (b), which is the horizontal gradient of diabatic heating. A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
Impacts on vertical vorticity development of the horizontal non-uniformity of diabatic heating

According to Eq. (2.16), the horizontal gradient of diabatic heating may contribute to the development of the vertical vorticity of a vortex, as indicated by Liu et al. (2001). There is a positive (negative) potential vorticity generation when the horizontal vorticity \( \mathbf{\eta} \) and the horizontal gradient of diabatic heating are in the same (opposite) direction; i.e. the potential vorticity generation is the largest (smallest or negative) when two vectors are parallel (orthogonal). As the horizontal variation of vertical velocity \( w \) is several orders of magnitude less than the vertical shear of horizontal wind \( \mathbf{V}_H \), Eq. (2.16) can be approximated as

\[
\left( \frac{DPV_e}{Dt} \right)_s = \mathbf{\eta}_s \cdot \nabla Q \approx p \left( \mathbf{k} \times \frac{\partial \mathbf{V}_H}{\partial z} \right) \cdot \nabla Q. \tag{2.17}
\]

This means that the sign of the change in \( PV_e \) depends on the disposition between the atmospheric circulation and diabatic heating. As illustrated schematically in Fig. 2b, the horizontal vorticity \( \mathbf{\eta} \) due to the vertical shear \( \left( \frac{\partial \mathbf{V}_H}{\partial z} \right) \) is perpendicular to, and pointing to the left of, the shear vector in the Northern Hemisphere, while the horizontal gradient of diabatic heating \( \left( \nabla Q \right) \) points toward the center of the diabatic heating. Consequently, on the right (left) side of the shear vector, the scalar product of the horizontal vorticity \( \mathbf{\eta} \) associated with the vertical shear \( \left( \frac{\partial \mathbf{V}_H}{\partial z} \right) \) of horizontal wind and the horizontal gradient of diabatic heating \( \left( \nabla Q \right) \); i.e. \( \mathbf{\eta}_s \cdot \nabla Q \), is positive (negative), corresponding to positive (negative) \( PV_e \) generation. According to Eq. (2.12), in a stable stratified atmosphere, positive and negative vorticities will be generated on the right and left sides, respectively, of the vertical shear of horizontal wind. In such circumstances, the horizontal gradient of diabatic heating can not only strengthen the vertical vorticity on the right side of the shear vector and weaken the vertical vorticity on the left side of the shear vector, but also further influence the movement of the vortex as the tendency of vertical vorticity generation is asymmetrically distributed on the two sides of the shear vector that pass through the center of diabatic heating, which is usually on the eastern side of the vortex.

### 2.2.3 Adiabatic vorticity development

**Slantwise vorticity development on a sloping isentropic surface**

Defining \( \beta \in \left[ 0, \frac{\pi}{2} \right] \) as the acute angle between \( \nabla \theta \) and the vertical direction, i.e. \( \tan \beta = \frac{\partial \theta}{\partial z} \), and defining \( \Lambda \in \left[ 0, \pi \right] \) as the angle between horizontal vorticity \( \mathbf{\eta}_s \) and baroclinity \( \theta_s \), then

\[
\begin{align*}
C_D &= \frac{\eta_s \partial \Lambda}{\theta_s} = +\eta_s \cos \Lambda \tan \beta, \quad \text{if } \theta_s > 0 \\
C_D &= \frac{\eta_s \partial \Lambda}{\theta_s} = -\eta_s \cos \Lambda \tan \beta, \quad \text{if } \theta_s < 0 \tag{2.18}
\end{align*}
\]

where \( \eta_s = |\mathbf{\eta}_s| \) and \( \theta_s = |\mathbf{\theta}_s| \). As

\[
\nabla \theta = \theta_s \mathbf{i} + \theta_s \mathbf{j} + \theta_s \mathbf{k} = \theta_s \mathbf{s} + \theta_s \mathbf{k} = \theta_s \mathbf{n},
\]

where \( \theta_s = |\nabla \theta| \), the unit vector \( \mathbf{n} \) is the direction of \( \nabla \theta \), and the unit vector \( \mathbf{s} \) is the direction of \( \nabla \theta \), then

\[
\begin{align*}
PV_e &= \mathbf{n} \cdot \nabla \theta = \eta_s \theta_s > 0, \Rightarrow \eta_s > 0, \quad \text{if } \theta_s > 0 \\
PV_e &= \mathbf{n} \cdot \nabla \theta = \eta_s \theta_s < 0, \Rightarrow \eta_s < 0, \quad \text{if } \theta_s < 0, \tag{2.19}
\end{align*}
\]

where \( \eta_s = |\mathbf{\eta}_s| \cdot \mathbf{n} \) is the projection of absolute vorticity \( \mathbf{\eta}_s \) on the direction of \( \mathbf{n} \). Therefore, Eq. (2.3) can be rewritten as

\[
\begin{align*}
\eta_z &= +\eta_s \cos \beta, \quad \text{if } \theta_s > 0 \\
\eta_z &= -\eta_s \cos \beta, \quad \text{if } \theta_s < 0 \tag{2.21}
\end{align*}
\]

If \( C_D < 0 \), Eq. (2.18) yields

\[
\begin{align*}
\cos \Lambda < 0, \Rightarrow \Lambda \in \left[ \frac{\pi}{2}, \pi \right], \quad \text{if } \theta_s > 0 \\
\cos \Lambda > 0, \Rightarrow \Lambda \in \left[ 0, \frac{\pi}{2} \right], \quad \text{if } \theta_s < 0 \tag{2.22}
\end{align*}
\]

As both \( \eta \) and \( \theta \) are positive, using Eqs. (2.22) and (2.20), Eq. (2.21) can be rewritten as

\[
\eta_z = \frac{|\eta_s|}{\cos \beta} + \eta_s \cos \Lambda \tan \beta. \tag{2.23}
\]

Under the constraint of \( C_D < 0 \), taking the derivative of Eq. (2.23) with respect to \( \beta \) leads to

\[
\frac{D \eta_z}{D \beta} = \frac{\eta_s |\sin \beta|}{\cos^2 \beta} + \frac{|\eta_s| \cos \Lambda}{\cos^2 \beta} + \frac{1}{D \theta_s} \frac{\eta_s |\cos \Lambda| \tan \beta}{\cos \beta} + \frac{D |\eta_s|}{D \beta} \frac{1}{\cos \beta} \cos \beta. \tag{2.24}
\]

The terms in the first parentheses on the right side of Eq. (2.24) are second-order positive infinite, but the terms in the second parentheses are first-order positive or negative infinite when \( \beta \) approaches \( \frac{\pi}{2} \). Hence

\[
\frac{D \eta_z}{D \beta} \to \infty, \quad \text{if } \beta \to \frac{\pi}{2}, \tag{2.25}
\]

which demonstrates that when an air particle is sliding down the concave slope or up the convex slope of an isentropic surface and \( C_D < 0 \), its vertical vorticity will increase rapidly.
Assuming the directions of horizontal vorticity \( \eta \) and baroclinity \( \theta \) are strictly opposite (identical) for a stable (unstable) atmosphere; i.e., \( \Lambda = \pi (\Lambda = 0) \) for \( \theta_s > 0 \) \( (\theta_s < 0) \), then \( C_D > 0 \); and assuming \( \theta_s \) is constant, then \( \eta_s \) is constant as \( PV_c = \eta_s \theta_s \) is conserved. Thus, Eq. (2.23) becomes

\[
\eta_z = \frac{|\eta_s|}{\cos \beta} + \eta_s \tan \beta \Rightarrow \eta_z \rightarrow \infty, \quad \text{if} \quad \beta \rightarrow \pi/2,
\]

which is illustrated schematically in Fig. 2.3 (Fig. 2.4) for the down-sliding SVD in the stable (unstable) atmosphere. The up-sliding SVD in the stable or unstable atmosphere is explained by Eq. (2.26). As there is no essential difference between the down-sliding and up-sliding SVD, the schematic diagram for the up-sliding SVD is not shown here.

An air particle at the initial position \( A^0 \) possesses zero \( C_D \) because the isentropic surface is horizontal. During the adiabatic movement of the particle, because the directions of horizontal vorticity \( \eta \) and baroclinity \( \theta \) are strictly opposite (identical) for a stable (unstable) atmosphere, when the particle moves from \( A^0 \) to \( A \) on the same isentropic surface, its vertical absolute vorticity \( \eta_z \) is increased under the conservation of \( PV_c \).

It is worth noting that the above analysis is just a qualitative demonstration of the SVD concept. Under the premise \( C_D < 0 \), vertical vorticity will increase dramatically when a particle is moving along a sharply tilted isentropic surface; i.e. \( \beta \) increases rapidly.

### Slantwise vorticity development from a Lagrangian perspective

The above SVD theory is based on isentropic coordinates with a priori assumption that the sloping angle \( \beta \) of the isentropic surface increases with time. However, as \( \frac{\theta_s}{\theta_s} = \frac{\theta_s}{\theta_s} \frac{\theta_s}{\theta_s} \), the change in the tilting angle \( \beta \) of the isentropic surface must also be analyzed. This is inconvenient and usually ignored in the application of SVD. For practical reasons, and to generalize the analysis, the SVD will be studied from a Lagrangian perspective.

### Vorticity development (VD)

Taking the derivative of Eq. (2.3) with respect to time under the constraint of the conservation of Ertel potential vorticity leads to

\[
\left( \frac{D\eta_z}{Dt} \right)_A = - PV_c \frac{D\theta_s}{D\eta_z} \frac{DC_D}{D\eta_s}, \quad \theta_s \neq 0, \quad (2.27)
\]

where \( \frac{\theta_s}{\theta_s} \) denotes the adiabatic development of vertical vorticity. Thus, the necessary and sufficient condition for vorticity development (VD) is

\[
\frac{DC_D}{D\eta_s} = - PV_c \frac{D\theta_s}{D\eta_z} \frac{DC_D}{D\eta_s}, \quad \theta_s \neq 0. \quad (2.28)
\]

This is the condition given in Eq. (2.6) by Wu and Liu (1997), but in a different form.

### Slantwise vorticity development (SVD)

If

\[
\frac{DC_D}{D\eta_s} < 0, \quad (2.29)
\]

### Fig. 2.3: Schematic diagram showing the development of vertical vorticity in a stable atmosphere caused by the slantwise sloping of the isentropic surface when the directions of horizontal vorticity \( \eta \) and baroclinity \( \theta \) are opposite. Initially, a particle is at position \( A^0 \) on the horizontal part of a \( \theta \) surface, and \( PV_c(= \eta_s \theta_s) \) is conserved. When it slides down the \( \theta \) surface at angle \( \beta \) to position \( A \), due to \( \eta_s = \frac{\eta_s}{\eta_s} + \eta_z \), the increase in \( \beta \) can result in the development of \( \eta_z \).

### Fig. 2.4: As Fig. 2.3, but for the unstable atmosphere case when the directions of horizontal vorticity \( \eta \) and baroclinity \( \theta \) are strictly parallel.
Eq. (2.27) can be written as
\[
\left(\frac{D\eta}{Dt}\right)_A = -PV_x \frac{D\theta}{Dt} \frac{Dz}{D\theta} + DC_D \frac{DC_D}{D\theta}, \quad \theta_z \neq 0.
\]  
(2.30)

If
\[
\frac{DC_D}{D\theta} < 0 < -\frac{PV_x \frac{D\theta}{Dt}}{\theta_z}, \quad \theta_z \neq 0,
\]
(2.31)

Eq. (2.30) yields
\[
\left(\frac{D\theta}{Dt}\right)_A \rightarrow \infty, \quad \text{if} \theta_z \rightarrow 0.
\]
(2.32)

This implies that when Eq. (2.31) is satisfied, the vertical vorticity of an air particle can develop rapidly if the atmosphere’s static stability approaches neutral.

In general, in an inertial stable atmosphere, the signs of \(PV_x\) and \(\theta_z\) are identical. In such circumstances, Eq. (2.31) can be interpreted as
\[
\begin{align*}
\frac{D\theta}{Dt} < 0 \text{ and } \frac{DC_D}{D\theta} < 0, & \quad \text{if } \theta_z > 0 \\
\frac{D\theta}{Dt} > 0 \text{ and } \frac{DC_D}{D\theta} < 0, & \quad \text{if } \theta_z < 0.
\end{align*}
\]
(2.33)

Therefore, the SVD can be stated as: when an air particle is sliding down the concave slope or up the convex slope of an isentropic surface in a stable stratified atmosphere with its static stability decreasing, or in an unstable atmosphere with its static stability increasing, its vertical vorticity can develop rapidly if its \(C_D\) is decreasing. That is, the vertical vorticity of an air particle will intensify rapidly when its stability approaches zero; i.e. the particle is inclined to neutral stability if its \(C_D\) decreases.

### Relationship between \(PV_x\) and SVD

The necessary and sufficient condition of Eq. (2.31) for SVD can be written as
\[
\frac{1}{\theta_z} \frac{DPV_x}{Dt} - \frac{PV_x \frac{D\theta}{Dt}}{\theta_z} \frac{Dz}{D\theta} < 0 < -\frac{PV_x \frac{D\theta}{Dt}}{\theta_z} \frac{Dz}{D\theta}, \quad \theta_z \neq 0.
\]
(2.34)

Adding a negative term \(\frac{PV_x}{\theta_z} \frac{D\theta}{Dt}\) to the inequality in Eq. (2.34) leads to
\[
-\frac{1}{\theta_z} \frac{DPV_x}{Dt} + \frac{\eta_z \frac{D\theta_z}{D\theta}}{\theta_z} > -\frac{PV_x \frac{D\theta}{Dt}}{\theta_z} \frac{Dz}{D\theta} > 0, \quad \theta_z \neq 0.
\]
(2.35)

For \(PV_x\) conservation, Eq. (2.5) becomes
\[
\left(\frac{Dz}{Dt}\right)_A = -\frac{1}{\theta_z} \frac{DPV_x}{Dt} - \frac{\eta_z \frac{D\theta_z}{D\theta}}{\theta_z} > -\frac{PV_x \frac{D\theta}{Dt}}{\theta_z} \frac{Dz}{D\theta} > 0, \quad \theta_z \neq 0.
\]
(2.36)

This provides the relationship between SVD and the change in \(PV_x\), and indicates that in a stable (unstable) atmosphere, a decrease (increase) of \(PV_x\) will result in vertical vorticity development. The intensity of SVD depends on the change of atmospheric static stability:
\[
\left(\frac{D\eta}{Dt}\right)_A > PV_x \frac{D}{D\theta} \left(\frac{1}{\theta_z}\right), \quad \theta_z \neq 0,
\]
(2.37)

which indicates that only when \(PV_x \frac{D}{D\theta} \frac{1}{\theta_z} > 0\) and the adiabatic development \(\frac{DC_D}{D\theta}\) of vertical vorticity development exceeds \(PV_x \frac{D}{D\theta} \frac{1}{\theta_z}\), can such SVD actually be defined as SVD. For a symmetric stable atmosphere with \(PV_x > 0\), when the static stability decreases to approach neutral stratification, SVD will occur, leading to the occurrence of severe weather.

### 2.3 Application of generalized slantwise vorticity development

#### 2.3.1 Data and computational method

**Data**

The data used in this study are the ERA-Interim reanalysis data with a horizontal resolution of 0.75° (Dee et al., 2011) and the TRMM 3B42 precipitation data with a three-hour interval and a 0.25° horizontal resolution (Huffman et al., 2007). The ERA-Interim reanalysis data have 60 hybrid levels in the vertical with the top level at 0.1 hPa, and have been archived at six-hourly intervals since 0000 UTC 1 January 1979.

**Computational method**

All of the computations in this study were performed at the model level, with vertical coordinates transforming so that the vertical and horizontal components of vorticity and gradient were strictly perpendicular and acoline, respectively. In addition, the results were interpolated onto the isobaric or isentropic surface to simplify the plotting of the figures.

The GSVD theory presented above is based on a Lagrangian perspective. For application to the corresponding diagnosis using the ERA-Interim reanalysis data with a time span of 6 h, we perform a Lagrangian conversion to calculate the material derivative. The Lagrangian change of any quantity \(q\) can be evaluated as
\[
\frac{Dq}{Dt} = \frac{q(r(t)) - q(r(t - \Delta t))}{\Delta t},
\]
(2.38)

where \(r(t)\) and \(r(t - \Delta t)\) are the arrival and departure position of a particle, respectively. The standard
iterative algorithm for backward trajectory in a semi-Lagrangian transport scheme is not accurate enough for a very large time step. Here, the large time step is divided into multiple smaller intervals, and in each small interval $\delta \tau$, the departure $r(\tau - \delta \tau)$, can be obtained by Taylor series expansion of $r(\tau - \delta \tau)$ about the arrival position $r(\tau)$ following the idea of McGregor (1993),

$$r(\tau - \delta \tau) = r(\tau) + \sum_{n=1}^{N} \frac{(-\delta \tau)^n}{n!} \frac{D^n r(\tau)}{D\tau^n} + O(\delta \tau^{N+1}).$$

(2.39)

In Cartesian coordinates, $r = r \cos \lambda \cos \phi + r \sin \lambda \cos \phi j + r \sin \phi k$, so the first derivative $\frac{Dr}{D\tau}$ required in Eq. (2.39) can be calculated analytically as

$$\frac{Dr(\tau)}{D\tau} = [\frac{u \sin \lambda - v \cos \lambda \sin \phi + w \cos \lambda \cos \phi}{r} \hat{j} + [\frac{u \cos \lambda - v \sin \lambda \sin \phi + w \sin \lambda \cos \phi}{r} \hat{j} + [\frac{\cos \phi + w \sin \phi}{r}] \hat{k}. \quad (2.40)$$

By taking $\frac{D^n r(\tau)}{D\tau^n}$ in Eq. (2.40) and dropping the higher-order terms $\frac{D^3 r}{D\tau^3}$ and $\frac{D^4 r}{D\tau^4}$, the second derivative $\frac{D^2 r}{D\tau^2}$ required in Eq. (2.39) can be approximated as

$$\frac{D^2 r(\tau)}{D\tau^2} \approx \left[ \frac{-2 uv}{r} \sin \lambda - \frac{2 vw}{r} \cos \lambda \sin \phi + \frac{2 uv}{r} \sin \lambda \tan \phi}{r \cos \phi} - \frac{u^2}{r \cos \phi} \cos \lambda - \frac{v^2}{r \cos \phi} \cos \lambda \cos \phi \right] \hat{j} + \left[ \frac{2 uv}{r} \cos \lambda - \frac{2 vw}{r} \sin \lambda \sin \phi - \frac{2 uv}{r} \cos \lambda \tan \phi}{r \cos \phi} - \frac{u^2}{r \cos \phi} \sin \lambda - \frac{v^2}{r \cos \phi} \sin \lambda \cos \phi \right] \hat{j} + \left[ \frac{2 vw}{r} \cos \phi - \frac{v^2}{r} \sin \phi \right] \hat{k}. \quad (2.41)$$

As the derivatives $\frac{Dr(\tau)}{D\tau}$ and $\frac{D^2 r}{D\tau^2}$ are obtained analytically, the algorithm of Eq. (2.39) is at a third-order precision and is more accurate than the numerical iterative algorithm. Therefore, Eq. (2.38) was used to calculate the Lagrangian change of various variables below.

### 2.3.2 Description of the TPV in 2008

#### Track of the TPV and associated precipitation

On 20 July 2008, a TPV moved eastward over the eastern TP (Fig. 2.5). It moved out of the TP after 1800 UTC 20 July 2008 before shifting suddenly southwards at 1200 UTC 21 July 2008. Finally, it moved east-northeastwards from 22 July 2008 and died out over the Yellow Sea on 23 July 2008. The track of the TPV and the horizontal pattern of associated 6-hour accumulated precipitation are shown in Fig. 2.5, and the time series of the central maximum vertical relative vorticity and the $3^\circ \times 3^\circ$ area-averaged of every 6-hour precipitation around the center of the TPV are shown in Fig. 2.6. As the TPV moved eastward from 1800 UTC 20 July 2008 onwards, it generated heavy rainfall over eastern China to its east, especially over the Sichuan Basin, and Hubei, Anhui, Henan, and Shandong provinces. As shown in Fig. 2.6, the central vertical relative vorticity intensified at 1800 UTC 20 July 2008 as the TPV began to move down the TP. On 21 July 2008, the TPV was located over the Sichuan Basin, and while the vertical relative vorticity decreased, precipitation over the center and east of the Sichuan Basin increased tremendously. The period from 0000 UTC 20 to 1800 UTC 21 July 2008 is defined as the first stage of the TPV development, during which it slid down from the TP into the Sichuan Basin and triggered severe rainfall over Sichuan Province. The second stage of the TPV is from 0000 UTC 22 to 1800 UTC 23 July 2008, during which the TPV moved northeastward and produced heavy rainfall over Hubei, Anhui, Henan, and Shandong provinces. At this stage, the vertical vorticity strengthened again at 0600 UTC 22 July 2008, then weakened gradually. However, the intensification of precipitation occurred at 0000 UTC 22 July 2008, ahead of the intensification of vertical vorticity (Fig. 2.6). This phenomenon implies that the vertical vorticity intensification may be due to the diabatic heating associated with precipitation, which will be discussed further in Section 3.3.

#### Large-scale circulation associated with the TPV

The distribution of the 300 and 500 hPa horizontal wind, geopotential height, and vertical relative vorticity at 0600 and 1800 UTC 20 July 2008 are presented in Fig. 2.7a and 2.7b, respectively. This figure demonstrates the circulation associated with the TPV before it moved down off the TP. At 0600 UTC 20 July 2008 (Fig. 2.7a) at 300 hPa in mid latitudes, a weak geopotential height ridge was located near 100 $\degree$E, and a trough was located over northern China. Along the subtropics, westerly prevailed with a weak trough over the northeastern TP. At 500 hPa, the trough was near the TP surface and the circulation pattern was similar to that at 300 hPa; however, the trough over the northeastern TP was strong. A strong closed cyclone with vorticity of more than $10^{-4} s^{-1}$, which we defined as the TPV in the current study, was embedded in the trough. By 1800 UTC (Fig. 2.7b), the general features of the circulation had changed little, except for a shift to the east. Consequently, the eastern portion of the TPV had slid down the eastern slope of the TP, while the vorticity center was still over the eastern TP. Figure 2.7c and 2.7d are the same as Fig. 2.7a and 2.7b, but for 500 and 700 hPa at 0600 and 1200 UTC 22 July 2008, respectively, which
show the circulations associated with the vortex during its second stage. The circulation showed high geopotential to the west of 100°E and low geopotential to its east. As the TPV moved eastward, the closed cyclone slid down to the lower troposphere, and evolved into a developing vortex at 700 hPa and an eastward moving mid-latitude trough at 500 hPa. As described above, the TPV re-intensified at 0600 UTC 22 July 2008 (Fig. 2.7c), and reached 500 hPa where the closed contour of 580 dgpm was re-established at 1200 UTC 22 July 2008 (Fig. 2. 7d).

2.3.3 Diabatic vorticity development due to inhomogeneous heating

Relative contributions to vertical vorticity development of the changes in PV\(_1\), PV\(_2\), and \(\theta\):

By applying Eq. (2.38) to each term in Eq. (2.5), the change in vertical vorticity can be calculated for every time step. Figure 2.8 shows these relative contributions when the vortex slid down the TP; i.e. the relative contribution to the development of vertical vorticity due to the changes in PV\(_1\), PV\(_2\), and \(\theta\). It demonstrates that the contribution...
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Fig. 2.7: Distributions of wind (vector), geopotential height (dpam; contour), and vertical relative vorticity ($10^{-5}$ s$^{-1}$; shading) at (a) 300 and 500 hPa at 0600 UTC 20, (b) 300 and 500 hPa at 1800 UTC 20, (c) 500 and 700 hPa at 0600 UTC 22, and (d) 500 and 700 hPa at 1200 UTC 22 July 2008. A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
Due to the change of $\theta$ $z$ $DPV$ $e$ is analogous to the total change $\frac{D\eta}{Dt}$ of vertical vorticity and with the same order of magnitude. The positive center of the contribution $\frac{1}{\theta} \frac{DPV}{Dt}$ due to the change in $PV$ is coincident with the development center of vertical vorticity of the TPV, though weaker in magnitude compared with the impact of diabatic heating. On the other hand, the contribution $\frac{-\theta}{\theta} \frac{Du}{Tr}$ due to the change in static stability $\theta$ to the development of vertical vorticity is usually negative and is weaker in magnitude.
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magnitude; i.e. the vertical vorticity decreases when the static stability $\theta_e$ increases. In other words, in a stable atmosphere, when the atmosphere becomes more stable, it does not favor vortex development. This differs from the linear model of flow over a mountain (on the lee side, the air column is stretched and static stability $\theta_e$ decreases, so vertical vorticity develops). The increase in $\theta_e$ is due to the nonlinear effect of the flow around the TP and the diabatic heating, that is, decreases in elevation and nocturnal surface cooling weaken the low-level potential temperature while the mid-level condensation latent heating associated with precipitation increases potential temperature aloft, so the Lagrangian change of $\theta_e$ increases. The above analysis indicates that the Lagrangian change of $PV_e$ associated with diabatic heating plays a leading role in the development of vertical vorticity during the intense development of the TPV.

During the second stage of the TPV, at 0000 UTC 22 July 2008 (Fig. 2.9a), the vertical vorticity developed at low levels near 700 hPa, and weakened at middle levels near 500 hPa. At 0600 UTC (Fig. 2.9b), the vertical vorticity intensified at both low and middle levels, and the TPV extended above 500 hPa. At 1200 UTC (Fig. 2.9c), the development of vertical vorticity near 500 hPa was maintained, but that at 700 hPa began to weaken. Afterwards, the vertical vorticity at both low and middle levels weakened and moved gradually northeastward (figure omitted). The same pattern can be seen more clearly from the contribution due to the change in $PV_e$ (second column in Fig. 2.9). In addition, the contribution due to the change in the horizontal component $PV_e$ of $PV_e$ was also significant near 500 hPa at 0600 UTC (Fig. 2.9b). As during the first stage, the contribution due to the change in static stability $\theta_e$ was negative at 0000 UTC 22 July 2008 (Fig. 2.9a). However, Fig. 2.9b and 2.9c show that the contribution due to the change in static stability $\theta_e$ was positive surrounding the center of the vortex at the 330 K level. This indicates that in such circumstances, the static stability $\theta_e$ contributes positively to the development of vertical vorticity; i.e. when the static stability $\theta_e$ decreases (increases) in a stable (unstable) atmosphere, vertical vorticity will develop.

Effect of the vertical gradient of diabatic heating

The above results demonstrate that the change in $PV_e$ plays a leading role in the development of vertical vorticity. The change of $PV_e$ is due to friction and diabatic heating. According to Eq. (2.15), in the free atmosphere, the change in $PV_e$ is due to diabatic heating. To understand its impact, the total change in vertical vorticity ($\eta_z$), the change in $PV_e$, the diabatic heating $(\eta_z Q_{diab})$, and the effect of vertical gradient of diabatic heating $(\eta_z Q_{diab}/\theta_e)$ for the second stage are demonstrated in Fig. 2.10.

At 1800 UTC 21 July 2008, there was no significant diabatic heating at middle levels, and the low-level positive $PV_e$ generation by the vertical gradient of diabatic heating was weak, so the development of the low-level vortex was weak and the TPV was concentrated at low levels (figure omitted). Subsequently, when the TPV arrived at the up-slope of the northeastern edge of the Sichuan Basin, the diabatic heating strengthened and uplifted gradually from 0000 to 1200 UTC 22 July 2008 (Fig. 2.10). Correspondingly, the vertical vorticity in the lower troposphere near the center of the vortex was intensified, and the TPV developed vertically above 500 hPa, which is consistent with the intensification above 500 hPa presented in Figs. 2.7c, 2.7d, and 2.9. Figure 2.10 shows that there was positive $PV_e$ generation below the maximum of diabatic heating to strengthen the low-level vortex, and negative $PV_e$ generation above it to weaken the upper-level vortex.

Effect of the horizontal gradient of diabatic heating

It is worthwhile noting that in Fig. 2.10, there are distinct differences between $\eta_z Q_{diab}/\theta_e$ and $\eta_z Q_{diab}/\eta_s$. This implies that other factors, such as the horizontal gradient of diabatic heating, also contribute to the development of vertical vorticity under certain circumstances.

During different development stages of the TPV, the distribution of the $PV_e$ generation $(\eta_z \cdot \nabla Q)$ due to the horizontal gradient of diabatic heating, the diabatic heating $(Q)$, and the vertical shear $(\eta_z Q_{diab}/\eta_s)$ of horizontal wind are shown in Figs. 2.10 and 2.11. According to Eq. (2.16), the positive $PV_e$ generation in association with the intensification of vertical vorticity; i.e. $(\partial PV_e/\partial z) = \eta_z \cdot \nabla Q$, is mostly located on the right side of the vertical shear $(\partial V/\partial z)$ of horizontal wind across the center of diabatic heating. Although the magnitude of $PV_e$ generation due to the horizontal gradient of diabatic heating is commonly one order of magnitude less than that due to the vertical gradient of diabatic heating, sometimes it can reach the same order. Such a case developed after the vortex slid down the TP at 0600 UTC 21 July 2008 (Fig. 2.11c), and also when it was climbing up the hillside over the northeastern Sichuan Basin at 0600 UTC 22 July 2008 (Fig. 2.12b). It is evident from Figs. 2.11 and 2.12 that, in most cases, the movement of the vortex was toward positive $PV_e$ generation on the right side of the vertical shear of horizontal wind at 400 hPa due to the horizontal gradient of diabatic heating. For example, the sudden southward shifting of the track of the vortex at 1200 UTC 21 July 2008 (Fig. 2.11d) is well correlated with the fact that the positive $PV_e$ generation associated with the horizontal gradient of diabatic heating was located to its south. There was an exception at 1200 UTC 22 July (Fig. 2.12c) when the vortex had climbed up the hill and became weaker and stagnant above
Fig. 2.9: As Fig. 2.8, but on the 330 and 315 K isentropic surfaces at (a) 0000, (b) 0600, and (c) 1200 UTC 22 July 2008. A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
Even so, the maximum of $PV_e$ generation ($\mathbf{\eta} \cdot \nabla Q$) due to the horizontal gradient of diabatic heating at that moment was still on the eastern side of the TPV, which subsequently led to the eastward movement of the vortex. The relationship between the movement of the TPV and positive $PV_e$ generation due to the horizontal gradient of diabatic heating was more obvious at 400 hPa than at other levels (figure omitted).
Fig. 2.11: The change at 400 hPa of $PV_e$ (first column; $5 \times 10^{-2}$ PVU (6h)$^{-1}$) due to the horizontal gradient of diabatic heating (second column; $5 \times 10^{-1}$ K (6h)$^{-1}$) at (a) 1800 UTC 20, (b) 0000 UTC 21, (c) 0600 UTC 21, and (d) 1200 UTC 21 July 2008. The vector denotes vertical shear of horizontal wind. A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
2.3.4 Adiabatic vorticity development due to slantwise vorticity development

The above analysis demonstrates that diabatic heating plays a leading role in the development and movement of the TPV. However, at some stage in the evolution of the vortex, the adiabatic term \( \frac{1}{h} \frac{\partial \psi}{\partial z} \) plays a significant role in its development, which is comparable with the role of diabatic heating. An example of this situation occurred on 22 July 2008 when the vortex was climbing the upslope of the mountain range over northeastern Sichuan Province. As shown in Fig. 2.6, the area-average precipitation associated with the TPV reached a maximum of 13 mm in 6 h at 0000 UTC 22 July 2008, and the vertical relative vorticity of the TPV was intensified to a maximum \( 2 \times 10^{-1}\text{s}^{-1} \) at 0600 UTC 22 July 2008. The evolutions of vertical vorticity development (first column in Fig. 2.10) and
diabatic heating (third column in Fig. 2.10) of the TPV indicate that its intensity and precipitation developed rapidly during this 12-hour interval. More importantly, the contribution to this development from the diabatic heating\((\frac{1}{\theta} \frac{D PV}{\theta z} \gamma)\), second column in Fig. 2.10) was significant, with its intensity at each time step being close to the total change \(\frac{D \eta z}{Dt}\) in vertical vorticity. Besides, the height of its maximum center also increased from 2 km at 1800 UTC 21 July 2008 to 3 km at 0000 UTC 22 July 2008, and reached 4 km 6 hours later, so contributing to the vertical extension of the vortex.

Figure 2.13 shows the distribution on the 330 and 315 K isentropic surfaces of the terms \(\frac{D C_D}{Dt} = \frac{1}{\theta} \frac{D PV}{\theta z} - \frac{PV}{\theta z} \frac{D \theta}{Dt}\), \(\gamma = -\frac{PV}{\theta z} \frac{D \theta}{Dt}\), and \(\frac{D \eta z}{Dt}\). The cross indicates the location of the TPV. (a) 0000 UTC and (b) 0600 UTC on July 22, 2008. A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
UTC 22 July 2008. At 0000 UTC, in the region surrounding the vortex center on both the 330 and 315 K isentropic surfaces, $\frac{DC0}{z} > \gamma$ and $\left(\frac{DPV}{z}\right)_A < 0$, implying that the criteria in Eq. (2.28) for VD were not satisfied, and the vortex development at 0000 UTC was basically due to the diabatic heating. At 0600 UTC on the 315 K isentropic surface at the center of the vortex, $\frac{DC0}{z} < \gamma$ and $\left(\frac{DPV}{z}\right)_A > 0$, implying that the criteria in Eq. (2.28) for VD were satisfied, and the development of the vortex in the lower layer was at least partly due to the adiabatic contribution. However, as $\gamma > 0$ at this level, the SVD criteria in Eq. (2.31) are violated, so the adiabatic development of the vortex should be rather limited. On the 330 K isentropic surface at the center of the vortex, not only was $\frac{DC0}{z} > \gamma$, but also $\frac{DC0}{z} < 0 < \gamma$. This indicates that the criteria in Eqs. (2.28) and (2.31) were satisfied, thus both VD and SVD occurred. This is consistent with the upward intensification of the TPV at 0600 UTC 22 July 2008, as presented in Fig. 2.7.

As shown in Eq. (2.36), the vertical vorticity development $\left(\frac{DPV}{z}\right)_A$ under adiabatic conditions is composed of two parts: the part $\left(-\frac{1}{\eta} \frac{DPV}{z}\right)$ due to the change in the horizontal component $\left(PV_2\right)$ of potential vorticity, and the part $\left(-\frac{D}{\eta} \frac{DPV}{z}\right)$ due to the change in static stability. The part of the adiabatic development of vertical vorticity due to the change in $PV_2$ can be expressed as

$$\left(\frac{DPV}{z}\right)_{PV_2} = \frac{1}{\eta} \frac{DPV_2}{z} = -\frac{1}{\eta} \frac{D}{\eta} \left(\frac{\eta \cdot \theta}{z}\right) = -\frac{D}{\eta} \frac{\eta \cdot \theta}{z} \frac{\eta \cdot \theta}{z} \frac{D}{\eta} \left(\frac{\eta \cdot \theta}{z}\right) \quad (2.42)$$

The second, third, and fourth columns in Fig. 2.14 demonstrate the spatial distributions of these three terms in Eq. (2.42) on the 330 and 315 K isentropic surfaces on 22 July 2008. It is evident that both of the contributions to the development of vertical vorticity due to the changes in horizontal vorticity $\eta$, and baroclinity $\theta$, are positive. As shown in Fig. 2.1, $PV_2 = \eta \cdot \theta$, is commonly negative; that is, the directions of $\eta$ and $\theta$ are opposite. The positive contributions of the two terms on the right-hand side of Eq. (2.42) imply that the change in $\eta$ is opposite to the direction of $\theta$, and the change in $\theta$ is opposite to the direction of $\eta$, during the development of vertical vorticity. Thus, either the component of horizontal vorticity or that of baroclinity along the projected direction is increasing when vertical vorticity develops. This confirms the conclusion of Wu and Liu (1997) that the development of vertical vorticity is not necessarily associated with conversion of the vorticity from its horizontal to vertical component caused by the uneven lifting of a vortex. In other words, an increase in vertical vorticity may not require a decrease in horizontal vorticity, and can also be achieved by an increase in either horizontal vorticity or baroclinity along the projected direction. In summary, both the dynamical effect of horizontal vorticity, and the thermal effect of baroclinicity, make positive contributions to the development of vertical vorticity.

### 2.4 Discussion and conclusions

Based on the Lagrangian change equation of vertical vorticity deduced from the equation of the three-dimensional $PV_c$, the development and movement of a vortex due to diabatic and adiabatic processes were studied, from both a $PV-Q$ and a $PV-\theta$ perspective. This GVSD theory was used to diagnose the development and movement of a TPV that occurred in July 2008.

In most cases, the atmosphere below the tropopause is statically stable and under geostrophic balance, $C_D$ and $PV_2$ are usually negative. Thus, merely using $C_D < 0$ as a condition for SVD on an isentropic surface is incomplete, and although an increase in the tilting angle $\beta$ of an isentropic surface is required, it is inconvenient for operational and practical purposes. A generalized slantwise vorticity development was introduced to consider both diabatic and adiabatic processes from a Lagrangian perspective, and can be used in any coordinate system. This GSVD indicates that the non-uniformity of diabatic heating contributes not only to the intensification of vertical vorticity, but also to the movement of the vortex. The criteria for VD and SVD were developed to distinguish between the two: VD requires $\frac{DC0}{z} < \gamma$, in which $\gamma = -\frac{PV2}{\eta} \frac{\eta \cdot \theta}{z} \frac{\eta \cdot \theta}{z} \frac{D}{\eta} \left(\frac{\eta \cdot \theta}{z}\right)$, whereas SVD requires $\frac{DC0}{z} < 0 < \gamma$. This demonstrates that the demand for SVD is much more restricted than that for VD; correspondingly, $\frac{DPV}{z} > 0$ under VD, whereas $\frac{DPV}{z} > PV_c \frac{D}{\eta} \left(\frac{1}{\eta}\right)$ under SVD. This account for the frequent occurrences of frontal and cyclonic systems, and the scarcity of severe weather. When an air particle is sliding down the concave slope or up the convex slope of an isentropic surface in the stable (unstable) atmosphere with its static stability decreasing (increasing), its vertical vorticity can develop rapidly if $C_D$ is decreasing. That is, the mean vorticity of an air particle will intensify rapidly when its static stability approaches zero; i.e. the particle is inclined to neutral stratification, if $C_D$ decreases. The intensity of vertical vorticity development due to SVD can be estimated as $\frac{DPV}{z} > PV_c \frac{D}{\eta} \left(\frac{1}{\eta}\right)$, which indicates that when the atmosphere approaches neutral stratification, the development of vertical vorticity approaches infinity.
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The relative contribution to the development of vertical vorticity due to the change in $PV_e$, $PV_z$, and $\theta_z$ demonstrates that in a static stable atmosphere, the Lagrangian change of $PV_e$ associated with diabatic heating plays a leading role; the change of $PV_z$ has a positive, but less significant contribution; and the change of $\theta_z$ usually has a negative effect in a stable environment as the atmosphere becomes more stable. However, in some cases surrounding the center of the vortex, the vertical vorticity develops rapidly as the static stability decreases (increases) in the stable (unstable) atmosphere. These observations indicate that strong stable and unstable conditions alone are not indicative for the development of a vortex, while near neutral stratification is favorable for the development of a vortex; i.e. the vertical vorticity will develop rapidly when $\theta_z \to 0$. The vertical gradient of the diabatic heating creates positive (negative) $PV_e$ generation that strengthens (weakens) the vertical vorticity below (above) the maximum of diabatic heating. The re-intensification of the TPV, the vertical extent of which extends above 500 hPa during its second stage, is mainly due to the re-strengthening and the vertical uplifting of the diabatic heating. As condensation heating usually occurs on the eastern side of a vortex, the effect of the vertical non-uniform heating is to move the vortex eastwards, as well as to intensify the local vertical vorticity. The horizontal gradient of diabatic heating causes positive (negative) $PV_e$ generation on the right (left) side of the vertical shear of horizontal wind. The horizontal non-uniform heating not only intensifies the vertical vorticity on the right side of the vertical shear of horizontal wind, but also leads to the movement of the vortex towards the positive $PV_e$ generation site.

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Fig. 2.14: Contribution to the development of vertical vorticity (first column; $10^{-5}$ m$^2$ (kg s$^{-1}$ h$^{-1}$)) due to the change in $PV_z$ (second column; $0.5 \times 10^{-5}$ m$^2$ (kg s$^{-1}$ h$^{-1}$)), the change in $\eta$ (third column; $0.5 \times 10^{-5}$ m$^2$ (kg s$^{-1}$ h$^{-1}$)), and the change in $\theta$ (fourth column; $0.5 \times 10^{-5}$ m$^2$ (kg s$^{-1}$ h$^{-1}$)). A black and white version of this figure will appear in some formats. For the colour version, please refer to the plate section.
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At some stages of the TPV, the VD criteria are violated, implying that the vortex development depends solely on the diabatic heating. At other stages, the VD criteria are satisfied while the SVD criteria are violated, and the adiabatic development of the vortex is limited. At one point; i.e. 0600 UTC 22 July, on the 330 K isentropic surface, both the VD and SVD criteria were satisfied, and the adiabatic intensification of vertical vorticity contributed to the upward development of the vortex. It was further demonstrated that the change in $PV_z$ contributed to the intensification of the TPV from 0000 TO 0600 UTC 22 July 2008, when it slid up the up-slope of the northeastern mountain in the Sichuan Basin, because the changes in both horizontal vorticity and baroclinicity had a positive effect on the development of vertical vorticity.

The appearance of the strong signals concerning VD and SVD in the region surrounding the vortex compared with other parts of the troposphere indicates that GSVD can serve as a useful tool for diagnosing the development of severe weather systems. As the SVD has been extended to a saturated moist atmosphere (Wu et al., 1995) and can be extended to an unsaturated moist atmosphere (Gao et al., 2004), GSVD should be extended to a moist atmosphere in future to investigate the mechanisms that drive the formation and development of severe weather, such as the torrential rain in summer.

Diabatic heating plays a leading role in the development of vertical vorticity on most occasions, and static stability plays a significant role in the adiabatic development of vertical vorticity under the constraint of conservation of $PV_z$. In the real atmosphere, diabatic heating and static stability can interact in a complex feedback manner. It will be both important and challenging to further investigate the effect of this feedback on the development of severe weather systems. In addition, ongoing study of the relationship between the horizontal movement and horizontal gradient of diabatic heating, and its robustness, is required for verification. Also, the formation of the TPV, which was not considered in this study, is an interesting topic that is worthy of thorough investigation.

References


